MATH 140A Review: Inequalities and Absolute Values

1. Prove that if $0 < a_1 < a_2$ and $0 < b_1 < b_2$, then $a_1 \cdot b_1 < a_2 \cdot b_2$.

2. Let n be a natural number. Determine for what values of n the following holds

$$\left| \frac{n-1}{3n+1} - \frac{1}{3} \right| < \frac{1}{2020}.$$

3. We say that a function f is *increasing* if $f(x) < f(y)$ when $x < y$. We say that f is *decreasing* if $-f$ is increasing. Let $f(x) = \frac{1}{x^p}$ for $x > 0$. Without taking the derivative, determine for what values of $p \in (-\infty, \infty)$ the function f is decreasing on $x > 0$ and increasing on $x > 0$.